TEMPERATURE FIELD OF A SPHERICAL THERMOACOUSTIC RECEIVER

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An approximate solution is presented for the temperature field of a spherical thermoacoustic receiver with internal heat sources. For certain dimensions of the thermal sensor, the heating due to the absorption of the field of waves diffracted at the sensor must be taken into account when calculating the frequency dependence of the sensitivity of the spherical receiver.

Thermoacoustic receivers used for measuring the strength of sound vibrations operate on the principle of converting acoustic energy into thermal [1, 2].

Let us consider a spherical thermoacoustic receiver comprising a temperature-sensitive element (thermocouple, semiconducting thermoresistance, etc.) covered with a sound-absorbing material. In order to be specific, we may take a thermal element of the differential type (containing two sensors) as an example of a temperature-measuring device (Fig. 1).

A block diagram representing the measurement of acoustic intensity in a liquid bath 1 is shown in Fig. 2. At a certain distance from the source of the acoustic vibrations 3, lying outside the zone of the thermal boundary layer, is the hot sensor 2 of the differential thermoacoustic receiver; the cold sensor 4 is placed outside the zone of acoustic action, so that its temperature is equal to that of the surrounding medium. The voltage at the output of the thermoacoustic receiver is recorded by the indicator 5.

We shall characterize the thermoacoustic receiver by a sensitivity

$$\gamma = \frac{U}{I}\,. \tag{1}$$

The voltage U is related to the temperature difference $T_1 - T_0$ between the hot and cold sensors by the relation

$$U = \eta\,(T_1 - T_0). \tag{2}$$

The quantity η characterizing the properties of the bimetal connection may be found from the equation

$$\eta = \frac{\sigma}{e}\ln\frac{n_1}{n_2}\,.$$

Thus the calculation of sensitivity amounts to a determination of the temperature difference $T_1 - T_0 = (T_1 - T_{sur}) + (T_{sur} - T_0)$ between the hot and cold sensors, for which purpose we require to find the temperature field both within the hot sensor of the thermoacoustic receiver $(T_1 - T_{sur})$ and also outside it $(T_{sur} - T_0)$.

Let the hot sensor of the spherical thermoacoustic receiver consist of a thermocouple with radius a and a spherical rubber layer of thickness $R - a$.

Let us neglect the losses of acoustic energy in the metal thermocouple by comparison with the losses in the sound-absorbing rubber layer. Then in the thermal respect the hot sensor of the thermoacoustic receiver may be considered as comprising two concentric spheres, of which the inner one contains no heat sources while the outer one is filled with heat sources having an output rate $Q(r, \theta, \varphi)$. We shall calculate

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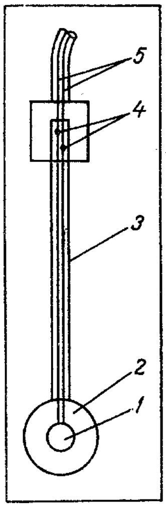

Fig. 1. Spherical thermoacoustic receiver: 1) thermocouple of the hot sensor; 2) sound-absorbing layer; 3) rubber tube for taking out the supply cables; 4) thermocouple of cold sensor; 5) cable feeding the thermal receiver.

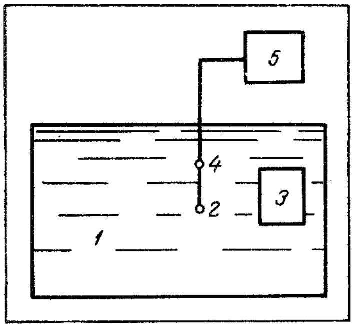

Fig. 2. Block diagram of the measuring system.

the temperature field in a spherical coordinate system r, θ, φ with its origin in the center of the hot sensor.

In order to determine the temperature differential $t = T_1 - T_{sur}$, we make use of the differential equation of heat conduction under steady thermal conditions, allowing for the internal heat sources [3]. Limiting consideration to the spherically-symmetrical case of heat transfer, in which the temperature is independent of φ and θ, and also to convective heat transfer without forced circulation (which is valid for operation in limited baths), the heat-conduction equation may be written in the following form

$$\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r} + \frac{1}{\lambda} Q(r) = 0. \tag{3}$$

Making a change of variables $T = \tau(r)/r$ we obtain

$$\frac{\partial^2 \tau}{\partial r^2} = -\frac{r}{\lambda} Q(r), \tag{4}$$

which on integration yields

$$T = -\frac{1}{r}\int\left[\int \frac{1}{\lambda} Q(r)\, r dr + C'\right] dr + \frac{C''}{r}. \tag{5}$$

Let us consider a thermal receiver in the form of a composite medium consisting of a spherical layer of sound-absorbing material (rubber) and an elastic spherical inclusion (thermocouple junction). Then the absorption of acoustic power in the inside of the thermal receiver determined by the output of the internal heat sources may be characterized by the absorption of the waves falling on the thermoacoustic receiver and the absorption of the diffracted sound waves [4]. In order to determine the optimum radii of the thermocouple and sound-absorbing layer corresponding to the maximum sensitivity of the thermoacoustic receiver, it is convenient to calculate the effects due to the absorption of these waves separately.

The output of the internal heat sources Q(r) is related to the total acoustic power within the thermoacoustic receiver by the following equation

$$Q(r) = Q_{\text{inc}}(r) + Q_{\text{dif}}(r) = \frac{d(\overline{W}_{\text{inc}} + \overline{W}_{\text{dif}})}{4\pi r^2 dr}. \tag{6}$$

Neglecting the reflections from the absorbing layer/liquid interface (in view of the small difference between the wave impedances of the liquid and the rubber), the scattering field associated with the conduits leading from the thermocouple, and also absorption in the thermocouple itself, we may determine the total flow of energy in terms of the displacements and mechanical stresses developing in the sound-absorbing layer of the spherical thermoacoustic receiver.

The solution to a problem of this kind was presented in [4] for an inclusion in the form of a hollow sphere. Carrying out an analogous discussion for an elastic medium, and allowing for the numerical relationships between the parameters of rubber-like and metallic media, after certain transformations appropriate to the case of Rayleigh scattering, we obtain the following expressions for the components $\overline{W}_{inc}$ and $\overline{W}_{dif}$ of the energy flow averaged over the period in question

$$\overline{W}_{inc} = \frac{8}{3} \pi r^3 \beta I, \tag{7}$$

$$\overline{W}_{dif} = \frac{0.46}{k_0^4 r} (k_0 a)^6 \left(\frac{\lambda_0}{\mu_0} + 2 \frac{q_0^2}{k_0^2} \right) \frac{\eta_\mu}{\eta_c} \left(1 - \frac{\rho}{\rho_1} \right)^2 \beta I. \tag{8}$$

Substituting Eqs. (7) and (8) into Eq. (6) we find

$$Q_{inc}(r) = 2 I\beta, \tag{9}$$

$$Q_{dif}(r) = 2 I\beta E \frac{a^6}{r^4}, \tag{10}$$

where the coefficient

$$E = 0.018 \left(\frac{\lambda_0}{\mu_0} + 2 \frac{q_0^2}{k_0^2} \right) \frac{\eta_\mu k_0^2}{\eta_c} \left(1 - \frac{\rho}{\rho_1} \right)^2.$$

The integration constants C' and C" in Eq. (5) are determined from the boundary conditions, which assume the form

$$\left.\frac{dT}{d\bar{r}}\right|_{\bar{r}=1} = - \mathrm{Bi}\, T\Big|_{\bar{r}=1}; \quad \left.\frac{dT}{d\bar{r}}\right|_{\bar{r}=\bar{a}} = 0, \tag{11}$$

where the Biot criterion $\mathrm{Bi} = \alpha R/\lambda$.

In order to calculate the heat-transfer coefficient α we must determine the law of heat transfer at the outer surface of the absorbing medium. This may be done, for example, by using the nomogram for the law of motion of a liquid presented in [3].

In order to calculate the temperature difference $T_{inc}(r)$, in Eq. (5) we substitute the rate of output of the sources $Q_{inc}(r)$ given by Eq. (9):

$$T_{inc}(r) = -\frac{Q_{inc}}{6\lambda} r^2 + C_1 + \frac{C_2}{r}. \tag{12}$$

Using the boundary conditions (11) we find

$$C_1 = -2 \varepsilon \bar{a}^3; \quad C_2 = \varepsilon (1 + 2 \bar{a}^3) + \frac{2 \varepsilon (1 - \bar{a}^3)}{\mathrm{Bi}}, \tag{13}$$

where

$$\varepsilon = \frac{Q_{inc}}{6\lambda} R^2. \tag{14}$$

Substituting (13) into (12), we obtain the following expression for the heating of any point of the layer relative to the liquid:

$$T_{inc}(\bar{r}) = \varepsilon \left[(1 - \bar{r}) \left(1 + \bar{r} - \frac{2\bar{a}^3}{\bar{r}} \right) + \frac{1 - \bar{a}^3}{3\, \mathrm{Bi}} \right]. \tag{15}$$

Transforming Eq. (15) with due allowance for (9) and (14), we find an expression for the temperature differential created by the absorption of the incident waves:

$$T_{inc}(\bar{r}) = \frac{\beta I R^2}{3\lambda} \left[(1 - \bar{r}) \left(1 + \bar{r} - \frac{2\bar{a}^3}{\bar{r}} \right) + \frac{1 - \bar{a}^3}{3\, \mathrm{Bi}} \right]. \tag{16}$$

As $\mathrm{Bi} \to \infty$, corresponding to the measurement of very low intensities (under 3 W/cm^2), and $\bar{a} = 0$, we obtain the ordinary solution for the heating of a sphere with a uniform distribution of heat sources [3]

$$T_{sph} = \frac{Q}{6\lambda} (R^2 - r^2).$$

In order to calculate the temperature differential $T_{dif}(r)$, we substitute (10) into (5):

$$T_{dif}(r) = -\frac{1}{r}\int\left[\int \frac{2I\beta}{\lambda}\,\frac{E a^6}{r^3}\,dr\right]dr + C_3 + \frac{C_4}{r}. \tag{17}$$

Let us integrate (17), express it in dimensionless form, and determine the integration constants from the boundary conditions

$$C_3 = \frac{2\beta I}{\lambda}\,E\bar{a}^6 R^4\left[\frac{1}{2} - \frac{1}{\bar{a}} + \frac{\bar{a}-1}{\bar{a}\,\mathrm{Bi}}\right],\quad C_4 = \frac{2\beta I}{\lambda}\,E\bar{a}^5 R^4. \tag{18}$$

Then

$$T_{dif}(\bar{r}) = \frac{\beta I E R^4 \bar{a}^5}{\lambda}\left\{\frac{(1-\bar{r})\,[2\bar{r} - \bar{a}(1+\bar{r})]}{\bar{r}^2} + \frac{2(\bar{a}-1)}{\mathrm{Bi}}\right\}. \tag{19}$$

The temperature of the sensor $T(\bar{a})$ relative to the temperature of the liquid is given by the sum of two components: the temperature differential created by the absorption of the incident waves

$$T_{inc}(\bar{a}) = \frac{\beta I R^2}{3\lambda}\left[(1+2\bar{a})(1-\bar{a})^2 + \frac{1-\bar{a}^3}{3\,\mathrm{Bi}}\right], \tag{20}$$

and that created by the diffracted waves

$$T_{dif}(\bar{a}) = \frac{\beta I R^2}{3\lambda}\,3ER^2\left[\bar{a}^4(1-\bar{a})^2 + \frac{2(\bar{a}-1)}{\mathrm{Bi}}\,\bar{a}^5\right]. \tag{21}$$

For intensities of the sound signal less than 3 W/cm^2, such as are usually encountered in practical acoustic measurements in liquid baths, $\mathrm{Bi} = \infty$. The computing equations (20) and (21) then simplify. In this case the function $T_{dif}(\bar{a})$ has a maximum for $\bar{a} = 0.66$. Numerical analysis of Eqs. (20) and (21) shows that, for practically all sizes of thermal receivers usually employed (radius R of the order of a few mm or over), in the sonic frequency range

$$3ER^2\bar{a}^4\Big|_{\bar{a}=0.66} \gg (1+2\bar{a})\Big|_{\bar{a}=0.66},$$

i.e.,

$$T_{dif}(0.66) \gg T_{inc}(0.66).$$

Hence the maximum sensor temperature $T(\bar{a})$ is reached for $\bar{a} = 0.66$.

Substituting (20) and (21) into (10) with $\mathrm{Bi} = \infty$ and allowing for Eq. (2) and the equation $T_1 - T_0 = T_{inc}(\bar{a}) + T_{dif}(\bar{a})$, we obtained the following relationship for the sensitivity of the spherical thermal receiver:

$$\gamma = \eta\,\frac{R^2}{\lambda}\,\beta(1-\bar{a})^2\left(ER^2\bar{a}^4 + \frac{2\bar{a}+1}{3}\right). \tag{22}$$

The calculated values of the quantities γ_{inc} and γ_{dif}, which constitute components of the sensitivity γ, are shown in Fig. 3 as functions of the dimensionless radius.

We carried out a calculation for copper–Constantan thermocouples of various radii vulcanized in a layer of rubber of the IRP-1075 type. The continuous curves 1, 2, 3 were calculated for a frequency ω and the broken curves 4, 5, 6 for a frequency 0.2ω. We see from Fig. 3 that the heating of the sensor due to the absorption of the diffracted waves may, for certain dimensions of the scattering object, exceed the heating due to the absorption of the incident wave by a factor of several times.

Figure 3 also shows the experimental curve 7, after normalizing its maximum to the maximum of the calculated curve 3. The comparative calculated and experimental data presented in Fig. 3 are in excellent agreement and justify the proposed method of calculating the sensitivity of spherical thermoacoustic receivers.

The analytical expressions given for determining the sensitivity of thermal receivers containing thermocouples as temperature-sensitive elements may be used for any type of sensitive spherical element (including semiconducting types) and any sound-absorbing material.

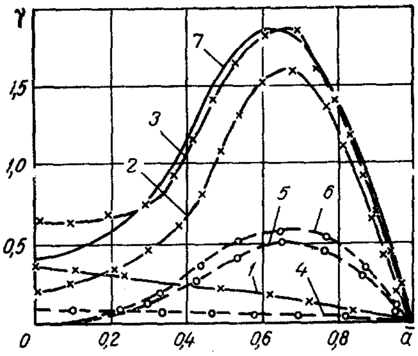

Fig. 3. Relation between the sensitivity of the thermoacoustic receiver and the dimensionless radius of the thermocouple (R = 0.3 cm): 1, 4) γ_{inc}; 2, 5) γ_{dif}; 3, 6) γ; 7) experiment; $\bar{a}$, dimensionless; γ in $mV \cdot cm^2/W$.

Thus our thermal calculations and experimental verification have shown that, in calculating the sensitivity of thermoacoustic receivers in which the dimensionless radius of the thermocouple $\bar{a}$ equals 0.3 or over, allowance should be made for the additional heating due to the absorption of waves diffracted at the thermocouple in the inside of the thermoacoustic receiver. The frequency characteristic of the sensitivity may be of a nonlinear nature.

NOTATION

U	is the output voltage of the thermoacoustic receiver;
I	is the intensity of the acoustic waves at the point of observation;
T_1 and T_0	are the average temperatures inside the hot and cold sensors of the thermoacoustic receiver, respectively;
T_{sur}	is the temperature on the surface of the thermoacoustic receiver;
σ	is the Boltzmann's constant;
e	is the charge on the electron;
n_1 and n_2	are the number of electrons in unit volume of each conductor, respectively;
λ	is the thermal conductivity of the material of the sound absorber;
t	is the temperature differential at the point of observation relative to the surface temperature of the thermoacoustic receiver;
T	is the current temperature;
$Q(r, \theta, \varphi)$	is the output of internal heat sources distributed over the volume of the thermoacoustic receiver;
r, θ, and φ	are the current coordinates in the spherical system;
R	is the radius of the hot sensor of the thermoacoustic receiver;
$\overline{W}_{inc}$ and $\overline{W}_{dif}$	are the time-averaged powers of the internal heat sources due to the absorption of the incident and diffracted waves, respectively;
β	is the pressure absorption coefficient in the sound-absorbing medium;
k_0 and q_0	are the real parts of the wave numbers of the longitudinal and transverse waves in the sound-absorbing material, respectively;
ρ and ρ_1	are the densities of the sound-absorbing medium and the sensor, respectively;
η_μ and η_c	are the loss coefficients associated with shear vibrations and velocity, respectively;
λ_0 and μ_0	are the real parts of the Lamé constants of the sound-absorbing medium;
$T_{inc}(\bar{r})$ and $T_{dif}(\bar{r})$	are the temperature differentials due to the absorption of the incident and diffracted waves, respectively, expressed as functions of the dimensionless radius of the sensor;
α	is the heat-transfer coefficient of the sound-absorbing material.

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